Bifurcations in Dynamical Systems: from classical towards random

Paulo Ruffino (ruffino@unicamp.br)
Mathematics Department
University of Campinas, Brazil

Abstract. We start with the observation that most bifurcation in the classical case is related with a topological break in the invariant measures of the system. With simple examples one can see the interesting phenomenon that in random systems – stochastic flows for instance, many of these 'breakings in the topology' can only be verified if one observes two or more, say n, simultaneous trajectories. This is the so called n-point motion in M^n, if M is the state space. A Markov chain with values in a finite space $M = \{1, \dots, m\}$, $m \geq 2$, has many different extensions to a compatible n-point Markov chain in M^n, for all $1 < n \leq m$. Embedding this phenomenon into the context of stochastic (Lévy) flows of diffeomorphisms in Euclidean spaces, we introduce the notion of an n-point bifurcation of a stochastic flow. Roughly speaking an n-point bifurcation takes place, when a small perturbation of the stochastic flow does not change the characteristics at lower level k-point motions, $k < n$, but does change at the level of n-point motion. We illustrate this phenomenon with an example of an n-point bifurcation, with $n \geq 3$. In addition, we present an algorithm for the detection of the precise level of an n-point bifurcation and a combinatorial formula for the dimension of the vector space of compatible extensions for flows of mappings on M. This is a joint work with Michael Högele.